Thornton & Marion 5th Edition Problem 2 – 43

A particle is under the influence of a force F = -kx + $\frac{k}{\alpha^2}x^3$ where k and α are constants and k is always positive. Determine the potential energy and discuss the motion. What happens when E = $\frac{1}{4}k\alpha^2$.

Define the function:

In[1]:= **Force[x_] = −k∗x + $\frac{k}{\alpha^2}$∗x^3;**

Force[x]

Out[2]= $-k\,x + \frac{k\,x^3}{\alpha^2}$

Integrate to find the potential energy:

In[3]:= **PotEnergy = −∫Force[x] ⅆx**

PotEnergy

Out[3]= $\frac{k\,x^2}{2} - \frac{k\,x^4}{4\,\alpha^2}$

Out[4]= $\frac{k\,x^2}{2} - \frac{k\,x^4}{4\,\alpha^2}$

Define the constants with random values:

In[5]:= **k = 2;**

α = 3;

Force[x]

PotEnergy

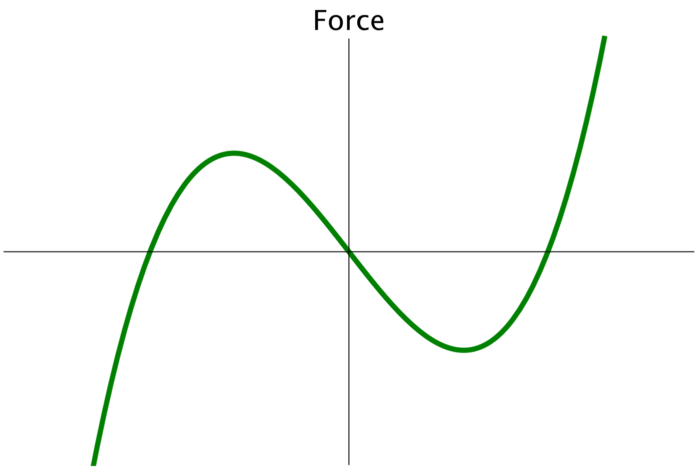
Out[7]= $-2\,x + \frac{2\,x^3}{9}$

Out[8]= $x^2 - \frac{x^4}{18}$

Plot the force.

In[ ]:=
```
pForce = Plot[Force[x], {x, -5, 5},
  BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
  Ticks → None, PlotRange → {-5, 5}, PlotPoints → 100,
  PlotStyle → {{RGBColor[0, 0.5, 0], Thickness[0.0075]}}, PlotLabel → "Force"]
```

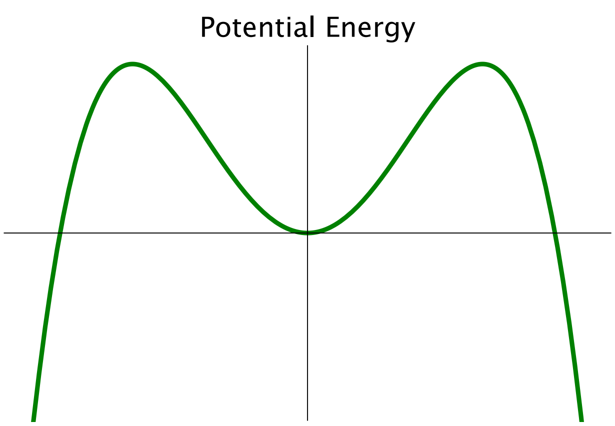
Out[ ]=



Plot U to show maxima and minima:

In[10]:=
```
pPotEnergy =
 Plot[PotEnergy, {x, -5, 5}, BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]}, Ticks → None,
  PlotRange → {-5, 5}, PlotPoints → 100, PlotStyle → {{RGBColor[0, 0.5, 0], Thickness[0.0075]}}, PlotLabel → "Potential Energy"]
```

Out[10]=



Find the maxima and minima of the potential energy. Since these occur where $\frac{dU}{dx} = -F = 0$, find the values for this.

In[11]:= Clear [k, α]

$$\text{Solve}\left[-k*x+\frac{k}{\alpha^2}*x^3==0,\ x\right]$$

Out[12]= {{x → 0}, {x → −α}, {x → α}}

The potential energy, $U(x) = \frac{k x^2}{2} - \frac{k x^4}{4 \alpha^2}$, at the maxima, $U(x_{max}) = U(\pm\alpha)$ and U(0) are

$U(\pm\alpha) = \frac{k \alpha^2}{2} - \frac{k \alpha^4}{4 \alpha^2} = \frac{k \alpha^2}{4}$ and

$U(0) = 0$.

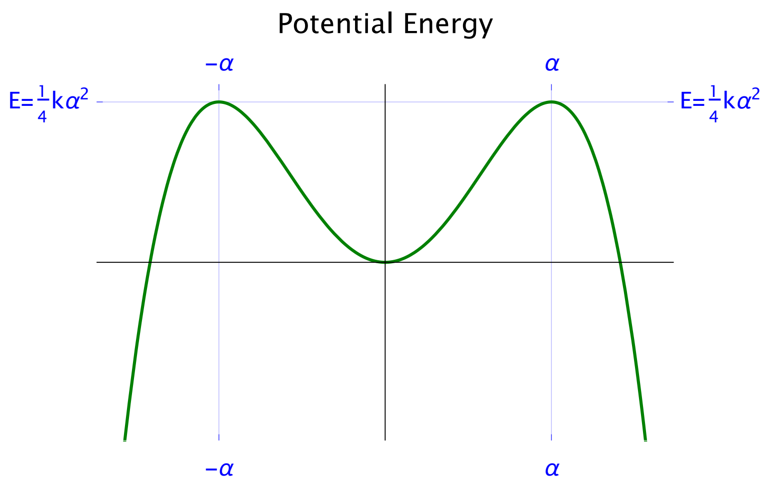
Thus when $E = T + U = \frac{k \alpha^2}{4}$, $E = U(\pm\alpha)$ so that T = O at these points. Thus the particle is bound as shown in the plot below for the values of k and α above.

```
In[13]:= k = 2;
         α = 3;
         k*α^2
         -----
           4

Out[15]= 9/2

In[16]:= p2PotEnergy = Plot[PotEnergy, {x, -5, 5}, BaseStyle → {FontFamily → Helvetica, FontSize → 12, FontColor → RGBColor[0, 0.5, 0]},
           PlotRange → {-5, 5}, PlotPoints → 20, Frame → True, FrameStyle → White,
           FrameTicks → {{{{4.5, "E=1/4 kα^2"}}, {{4.5, "E=1/4 kα^2"}}}, {{{-3, "-α"}, {3, "α"}}, {{-3, "-α"}, {3, "α"}}}},
           FrameTicksStyle → Directive[Blue], GridLines → {{{-3, Blue}, {3, Blue}}, {{4.5, Blue}}},
           PlotStyle → {RGBColor[0, 0.5, 0]}, PlotLabel → "Potential Energy"]
```

Out[16]=




```
In[18]:= Export["TM5P2_43.pdf", SelectedNotebook[]]

Out[17]= TM5P2_43.pdf
```